

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
International GCSE**

Centre Number

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Candidate Number

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Tuesday 15 January 2019

Morning (Time: 2 hours 30 minutes)

Paper Reference **4MB1/02**

Mathematics B

Paper 2



You must have: Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- **Calculators may be used.**

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Without sufficient working, correct answers may be awarded no marks.

Turn over ►

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Pearson

Answer ALL TWELVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

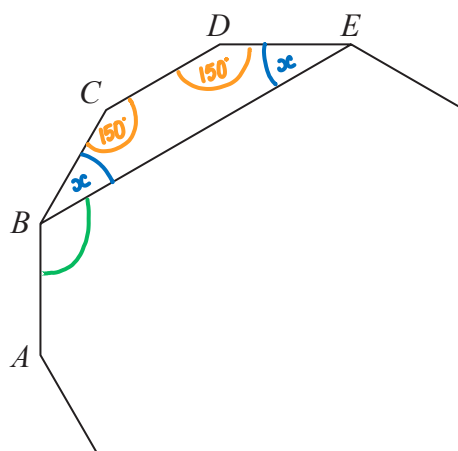


Diagram NOT
accurately drawn

Figure 1

Figure 1 shows part of a regular 12-sided polygon.

The vertices B and E of the polygon are joined with a straight line.

Calculate the size, in degrees, of $\angle ABE$.

Give reasons for each stage of your working.

In this question, you will first need to find the interior angle of the 12-sided polygon using the equation for the interior angle.

Equation for the interior angle of a regular polygon:

$$\frac{180(n-2)}{n}$$

Equation for the interior angle of a 12-sided polygon:

$$\begin{aligned}\frac{180(12-2)}{12} &= \frac{180 \times 10}{12} \\ &= \frac{1800}{12} \\ &= 150^\circ\end{aligned}$$

The shape $CBED$ is a parallelogram and as such, opposite angles BCD and CDE , and CBE and BED are equal.

We know that $\angle BCD$ and $\angle CDE$ are 150° since they are shared with the polygon.

Since all angles in a parallelogram add to 360° , you can find the remaining angles.

(Total for Question 1 is 4 marks)



$$\begin{array}{l} 2(150) + 2x = 360^\circ \\ \text{Expand} \rightarrow 300 + 2x = 360 \\ -300 \rightarrow 2x = 60 \\ \div 2 \rightarrow x = 30^\circ \end{array}$$

$$\angle CBE = 30^\circ$$

$$\angle ABE = \angle ABC - \angle CBE$$

$$\angle ABE = 150 - 30$$

$$\angle ABE = 120$$

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② $x + 6y = 15$

You can use two methods to solve these simultaneous equations:

Method 2: Substitution

① Make x the subject of equation ② and substitute into equation ①

$$\begin{array}{rcl} x + 6y & = & 15 \\ -6y & & \\ \hline x & = & 15 - 6y \end{array}$$

$$\begin{array}{r} x + 6y = 15 \\ \times 3 \quad \curvearrowright \\ 3x + 18y = 45 \end{array}$$

Since the signs of both coefficients of x are the same, you can subtract to eliminate x .

$$3x - 2y = 7$$

$$- \quad \underline{3x + 18y = 45}$$

$$-20y = -38$$

$y = 1.9$

② You can now substitute your value of y into one of the above equations to find x .

$$3x - 2(1.9) = 7$$

$$3x - 3.8 = 7$$

$$3x = 10.8$$

$x = 3.6$

$$3(15 - 6y) - 2y = 7$$

Expand $45 - 18y - 2y = 7$

Simplify $\rightarrow 45 - 20y = 7$

$$-20y = -38 \quad \swarrow$$

$y = 1.9$

② You can now substitute your value of y into equation you formed for x in part 1 and find x .

$$x = 15 - 6y$$

$$x = 15 - 6(1.9)$$

$$x = 15 - 11.4$$

$x = 3.6$



- 3 The original price of each 6-day ski pass is reduced by 15% in a sale.

In the sale the price of each 6-day ski pass is \$272

- (a) Calculate the original price of each 6-day ski pass.

(2)

The price of each 3-day ski pass is £110

The exchange rate is £1 = \$1.70

- (b) Calculate how much Andrew will save by buying one 6-day ski pass in the sale rather than two 3-day ski passes.

(3)

(a) There are two ways you can find the original price:

method 1:

If the full price is 100%, and it is reduced by 15%, the new

price is $(100 - 15) = 85\%$ of the original.

$$\begin{array}{l} \div 17 \quad \begin{array}{l} 85\% \rightarrow \$272 \\ \downarrow \\ 5\% \rightarrow \$16 \end{array} \quad \div 17 \\ \times 20 \quad \begin{array}{l} \uparrow \\ 100\% \rightarrow \$320 \end{array} \quad \times 20 \end{array}$$

method 2: the multiplier.

$$100\% - 15\% = 85\% = 0.85$$

$$\text{original price} \times 0.85 = 272$$

$$\begin{aligned} \text{original price} &= \frac{272}{0.85} \\ &= \$320 \end{aligned}$$

(b) Here, the cheaper of the two options will save Andy more money.

6-day ski pass trip cost \$272

You need to find out how much two ski-passes costs in dollars.

$$2 \times 110 = £220$$

Convert to dollars:

$$\begin{array}{l} \times 1.70 \\ \swarrow \\ £1 : \$1.70 \end{array}$$

$$\begin{array}{l} £220 : \$374 \\ \swarrow \\ \times 1.70 \end{array}$$

Compare the two prices:

$$374 - 272 = \$102$$

A 6 day pass will be \$102 cheaper.

(Total for Question 3 is 5 marks)



- 4 (a) Express 56 as the product of its prime factors.

(1)

Trains to Watson leave Denby station every 56 minutes.

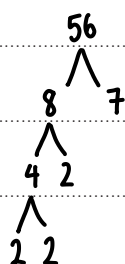
Trains to Barbe leave Denby station every 24 minutes.

A train to Watson and a train to Barbe both leave Denby station at 1200.

- (b) Find the next time that a train to Watson and a train to Barbe leave Denby station at the same time.

(3)

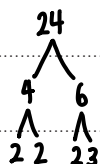
(a) You can draw a factor tree to help you:



$$56 = 2^3 \times 7$$

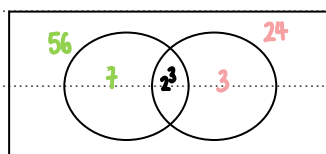
(b) To find the next time, you need the lowest common multiple of 56 and 24. You can find this by drawing a Venn diagram of all prime factors.

1. Express 24 as a product of its primes:



$$24 = 2^3 \times 3$$

2. Draw the Diagram



3. The LCM is the product of all prime factors:

$$\text{LCM} = 2^3 \times 7 \times 3$$

$$= 168$$

4. This tells us that the trains will leave together in 168 min.

168 min = 2 hours 48 min → You have to add this to 1200.

$$12.00 + 2.48 = 14.48$$

(Total for Question 4 is 4 marks)



- 5 (a) On the grid opposite, draw the graph of $y = 3x + 2$ for the values of x from 0 to 5 (2)
- (b) Show, by shading on the grid, the region R defined by all of the inequalities

$$y \leq 3x + 2 \quad \text{and} \quad y \geq 4 \quad \text{and} \quad 8 \leq 4x \leq 18$$

Label the region R .

(3)

(a) You need to substitute the values $x=0,1,2,3,4,5$ into the equation $y=3x+2$ to find the y -coordinates.

$$y_0 = 3(0) + 2 \quad y_1 = 3(1) + 2 \quad y_2 = 3(2) + 2 \quad y_3 = 3(3) + 2 \quad y_4 = 3(4) + 2 \quad y_5 = 3(5) + 2$$

$$y_0 = 2 \quad y_1 = 5 \quad y_2 = 8 \quad y_3 = 11 \quad y_4 = 14 \quad y_5 = 17$$

You can now plot these values on the graph below.

(b) If $y \leq 3x + 2$, all values of y are smaller than or equal to $3x + 2$.

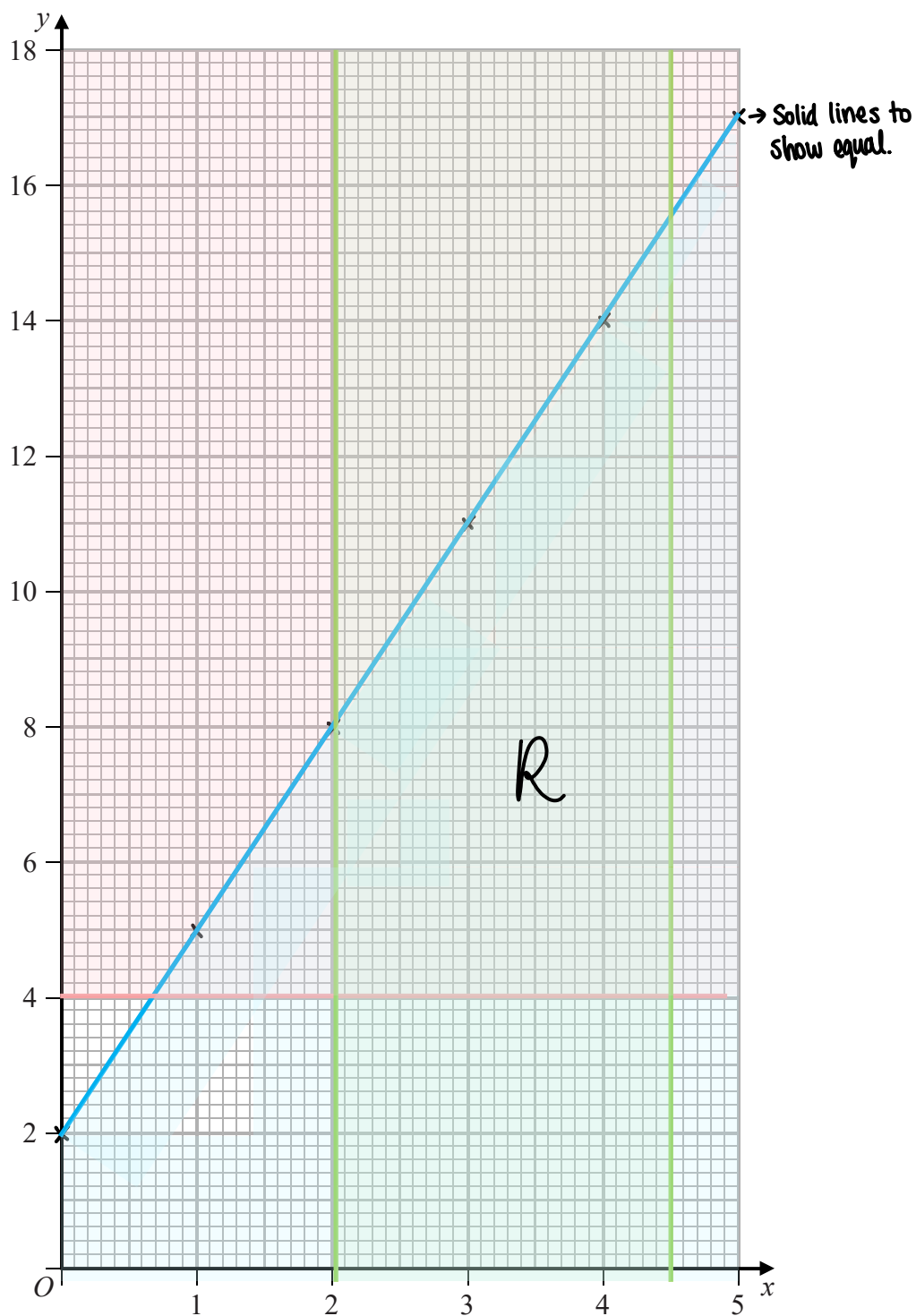
$y \geq 4$ mean all values of y are greater than or equal to 4.

$8 \leq 4x \leq 18 \rightarrow$ Here you have to divide all values by 4 to get x by itself
 $\div 4 \rightarrow 2 \leq x \leq 4.5 \rightarrow x$ is greater than or equal to 2 but less than or equal to 4.5

The area R is where all your values overlap if you shade in.



Question 5 continued



(Total for Question 5 is 5 marks)



- 6 One day, 80 customers at a health spa were asked if they used any of the gym (G), the pool (P) and the sauna (S).

Of these 80 customers

34 had used the gym

60 had used the pool

30 had used the sauna

20 had used the gym and the pool but not the sauna

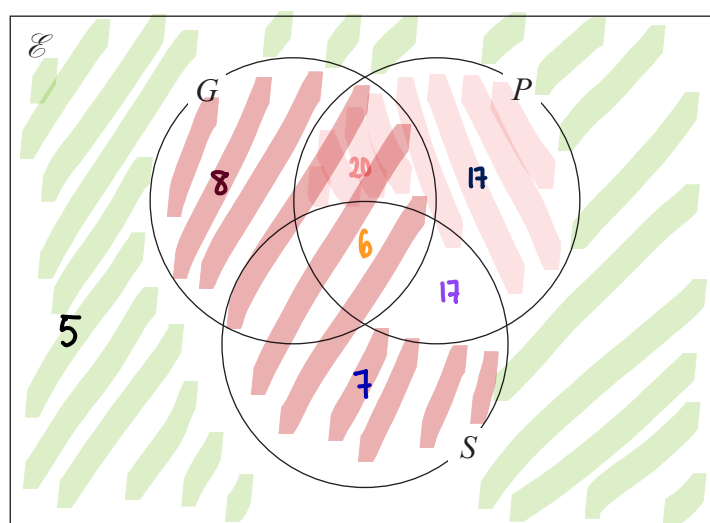
17 had used the pool and the sauna but not the gym

6 had used the gym, the pool and the sauna

no one had used the gym and the sauna but not the pool.

First label these values, then work out individually

- (a) Using this information, complete the Venn diagram to show the number of elements in each appropriate subset.



(3)

- (b) Find (i) $n([G \cup P \cup S]')$

(ii) $n(G \cup S)$

(iii) $n(P \cap S')$

Note:

$\cup$ = or / union

$\cap$ = and / intersection

(3)

(a) Only sauna $\rightarrow 30 - 6 - 17 = 7$

Only gym $\rightarrow 34 - 20 - 6 = 8$

Only pool $\rightarrow 60 - 20 - 6 - 17 = 17$

Next you must find out the number of people who did not use any of the facilities above:

$80 - 20 - 8 - 6 - 17 - 17 - 7 = 5$



Question 6 continued

(b) (i) $n([G \cup P \cup S]')$ refers to the number of people who did not use the gym or pool or sauna.

There were 5 people who did not use any.

Answer: 5

(ii) $n(G \cup S)$ means the number of people in G or S.

$$20 + 8 + 6 + 17 + 7 = 58$$

Answer: 58

(iii) $n(P \cap S')$ means the number of people in P but not S.

$$20 + 17 = 37$$

Answer = 37

(Total for Question 6 is 6 marks)



- 7 Vincent plays a game in which he can score 0, 1, 2, 3 or 4 each time he plays the game. The score he gets in a game is independent of the score he got in the previous games. The table gives information about the probability of getting each score in a game.

Score	0	1	2	3	4
Probability	x	$3x$	x	0.2	0.1

Vincent plays the game 150 times.

- (a) Calculate the number of times he would expect to score 1

(4)

Vincent plays the game another two times and records his two scores. He adds these two scores together to get his Total.

- (b) Calculate the probability that Vincent's Total is greater than 6

(3)

Given that for these two games Vincent has a Total greater than 6 points,

- (c) calculate the probability that he got a score of 4 in the first of these two games.

(3)

(a) For any of these questions, you first need to find the value of x .

All probabilities add up to 1.

$$x + 3x + x + 0.2 + 0.1 = 1$$

$$5x + 0.3 = 1$$

$$5x = 0.7$$

$$x = 0.14$$

The probability of 1 is: $3(0.14) = 0.42$

$$150 \times 0.42 = \underline{\underline{63}}$$

(b) The only way in which Vincent's scores are greater than 6 are if he scores a 3 and a 4 in either time or a score of 4 and 4 either time.

Probability of scoring a 3 and then a 4 $\rightarrow 0.2 \times 0.1 = 0.02$

Probability of scoring a 4 and then a 3 $\rightarrow 0.1 \times 0.2 = 0.02$

Probability of scoring a 4 and a 4 $\rightarrow 0.1 \times 0.1 = 0.01$

Total: $0.02 + 0.02 + 0.01 = \underline{\underline{0.05}}$



Question 7 continued

(c) Here you need to add the probabilities of both times Vincent got 4 first from part (b).

$$0.02 + 0.01 = 0.03$$

Next, divide by the total probability of a score greater than 6 for the two games:

$$\frac{0.03}{0.05} = 0.6$$

→ You get the 0.05 from part (b)!

(Total for Question 7 is 10 marks)



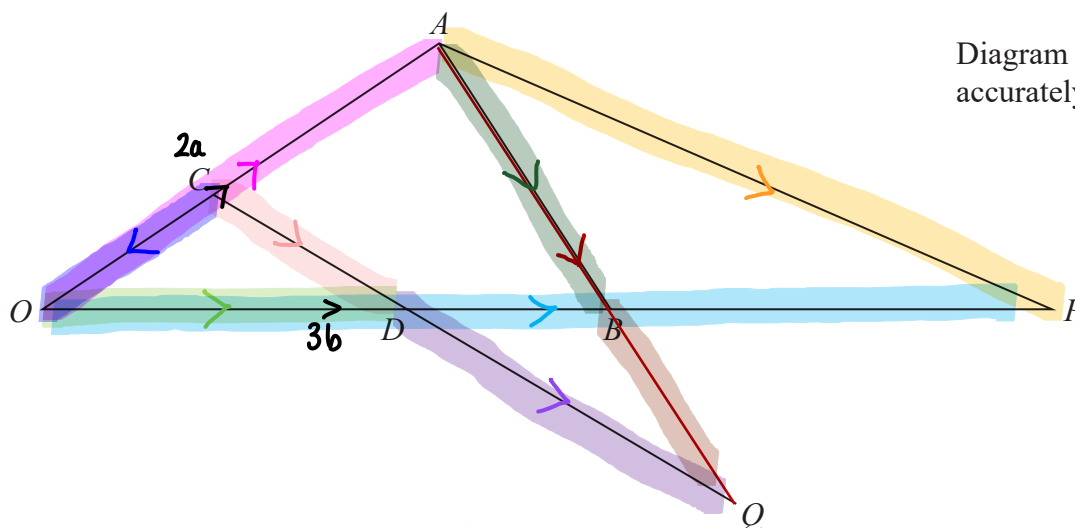


Figure 2

Figure 2 shows the triangle OAB with $\vec{OA} = 2\mathbf{a}$ and $\vec{OB} = 3\mathbf{b}$

The point C lies on OA such that $\vec{OC} = \frac{1}{3}\vec{OA}$

The point D lies on OB such that $\vec{OD} = \frac{2}{3}\vec{OB}$

(a) Find $\vec{CD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(2)

The point P is such that $ODBP$ is a straight line and AP is parallel to CD .

(b) Find $\vec{OP}$ in terms of $\mathbf{b}$.

(4)

The point Q is such that $\vec{CD} = \vec{DQ}$

(c) Show that A , B and Q are collinear.

(4)

(a) $\vec{CO} = \vec{CO} + \vec{OO}$ → You first need to find $\vec{OC}$ and $\vec{OD}$. $\vec{CO}$ is the negative of $\vec{OC}$ since it is in the opposite direction.

$\vec{OC} = \frac{1}{3}(2\mathbf{a}) = \frac{2}{3}\mathbf{a}$

$\vec{OD} = \frac{2}{3}(3\mathbf{b}) = 2\mathbf{b}$

$\vec{CD} = -\frac{2}{3}\mathbf{a} + 2\mathbf{b}$

(b) Since $\vec{AP}$ is parallel to $\vec{CD}$, it is a multiple of $\vec{CD}$.

$$\vec{CD} \times k = \vec{AP}$$

$$(-\frac{2}{3}\mathbf{a} + 2\mathbf{b}) \times k = \vec{AP}$$



Question 8 continued

$$\vec{OP} = \vec{OA} + \vec{AP}$$

$$\vec{OP} = (2a) + k(-\frac{2}{3}a + 2b)$$

$$\vec{OP} = 2a - \frac{2}{3}ka + k2b$$

Since we are solving in terms of b , the values of a will cancel out.

$$2a - \frac{2}{3}ka = 0$$

$$a(2 - \frac{2}{3}k) = 0$$

$$2 - \frac{2}{3}k = 0$$

$$-\frac{2}{3}k = -2$$

$$k = 3$$

$$\vec{OP} = (3)2b$$

$$= 6b$$

(c) Collinear means that A, B and Q are on a straight line.

For this, the lines $\vec{AB}$ and $\vec{BQ}$ must be factors of the line $\vec{AQ}$

$$\vec{AQ} = \vec{AO} + \vec{OQ} + \vec{OQ}$$

$$\vec{AB} = \vec{AO} + \vec{OB}$$

$$\vec{BQ} = \vec{BO} + \vec{OQ} \rightarrow \vec{BO} = \vec{BO} - \vec{OO} = 3b - 2b = b$$

$$\vec{OQ} = \vec{CO} = -\frac{2}{3}a + 2b$$

$$\vec{AQ} = -2a + 2b + (-\frac{2}{3}a + 2b)$$

$$= -\frac{8}{3}a + 4b$$

$$\vec{AB} = -2a + 3b$$

$$\vec{BQ} = b + (-\frac{2}{3}a + 2b)$$

$$= -\frac{2}{3}a + 3b$$

Now, you can compare to establish that they are collinear.



Question 8 continued

$$\vec{AQ} : \vec{AB}$$

$$\begin{array}{l} \text{Divide 'a' by} \\ -2 \end{array} \quad \begin{array}{l} -\frac{8}{3}a + 4b : -2a + 3b \\ \downarrow \\ \frac{4}{3}a + 4b : a + 3b \end{array} \quad \begin{array}{l} \text{Divide 'a' by} \\ -2 \end{array}$$

$$\begin{array}{l} \text{Divide 'b' by 3} \end{array} \quad \begin{array}{l} \frac{4}{3}a + \frac{4}{3}b : a + b \\ \downarrow \\ \frac{4}{3}a + \frac{4}{3}b : a + b \end{array} \quad \begin{array}{l} \text{Divide 'b' by 3} \end{array}$$

$$\quad \quad \quad \times \frac{4}{3}$$

$$\vec{AQ} = \frac{4}{3}\vec{AB}$$

$\therefore A, B$ and Q are collinear.

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Question 8 continued

Handwriting practice area with 20 horizontal dotted lines.

(Total for Question 8 is 10 marks)



9 The vertices of triangle A are the points with coordinates $(0, 0)$, $(1, 0)$ and $(0, 2)$

(a) On the grid opposite, draw and label triangle A .

(1)

Triangle A is transformed to triangle B under the transformation with matrix $\mathbf{M}$ where

$$\mathbf{M} = \begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix}$$

(b) On the grid, draw and label triangle B .

(3)

The transformation with matrix $\mathbf{M}$ is equivalent to an enlargement with centre the origin, with scale factor k followed by an anticlockwise rotation of θ° about the origin.

(c) Calculate the value of k .

(2)

(d) Calculate the value, to one decimal place, of θ .

(2)

Triangle B is transformed to triangle C under a reflection in the x -axis.

(e) On the grid, draw and label triangle C .

(1)

Triangle A is transformed to triangle C under the transformation with matrix $\mathbf{T}$.

(f) Find matrix $\mathbf{T}$.

(2)

(a) On graph

(b) You have to multiply each coordinate by the matrix and find the new coordinates.

Rule for multiplying a 1×2 by a 2×2 matrix:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

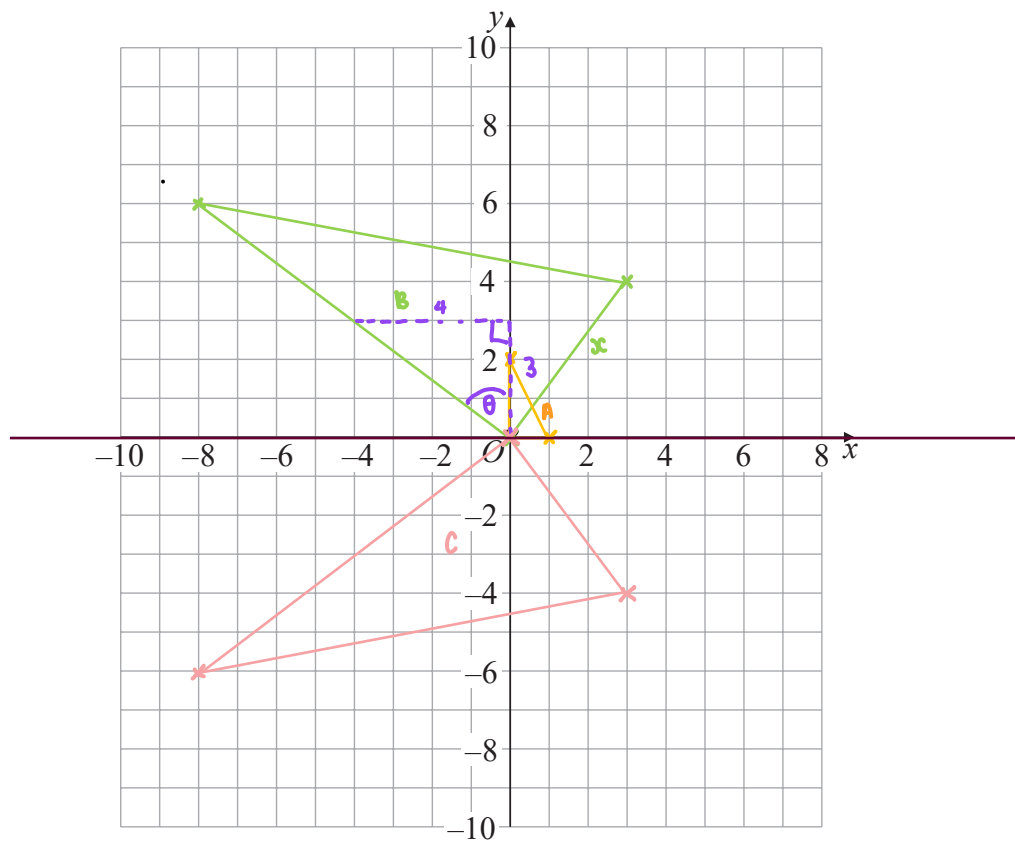
$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3(1) + -4(0) \\ 4(1) + 3(0) \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -4 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 3(0) + -4(2) \\ 4(0) + 3(2) \end{pmatrix} = \begin{pmatrix} -8 \\ 6 \end{pmatrix}$$

Plot these points and draw triangle B .



Question 9 continued



(c) To find k , you need to calculate the magnitude of one of the new lines compared to the old line.

Magnitude of $x_2 \Rightarrow \sqrt{3^2 + 4^2} = \sqrt{9 + 12} = 5$

Magnitude of $x \rightarrow 1$

$\therefore k = 5$

(d) You can find the angle θ using $S^{\circ}H C^A H T^{\circ}A$

You have the opposite and adjacent so you can use $T^{\circ}A$.

$$\tan \theta = \frac{\text{OPPOSITE}}{\text{ADJACENT}}$$

$$\tan \theta = \frac{4}{3}$$

$$\theta = \tan^{-1} \left(\frac{4}{3} \right)$$

$$\theta = 53.13 \dots$$

$$\theta \approx 53.1 \text{ (1 d.p.)}$$

Turn over for a spare grid if you need to redraw your triangles.



Question 9 continued

(e) Reflect on x -axis(f) Here you can work backwards to find T .

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} -8 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 0a + 2b \\ 0c + 2d \end{pmatrix} = \begin{pmatrix} -8 \\ -6 \end{pmatrix}$$

$$\begin{pmatrix} (1)a + (0)b \\ (1)c + (0)d \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

$$2b = -8$$

$$2d = -6$$

$$1a = 3$$

$$1c = -4$$

$$b = -4$$

$$d = -3$$

$$a = 3$$

$$c = -4$$

$$T = \begin{pmatrix} 3 & -4 \\ -4 & -3 \end{pmatrix}$$



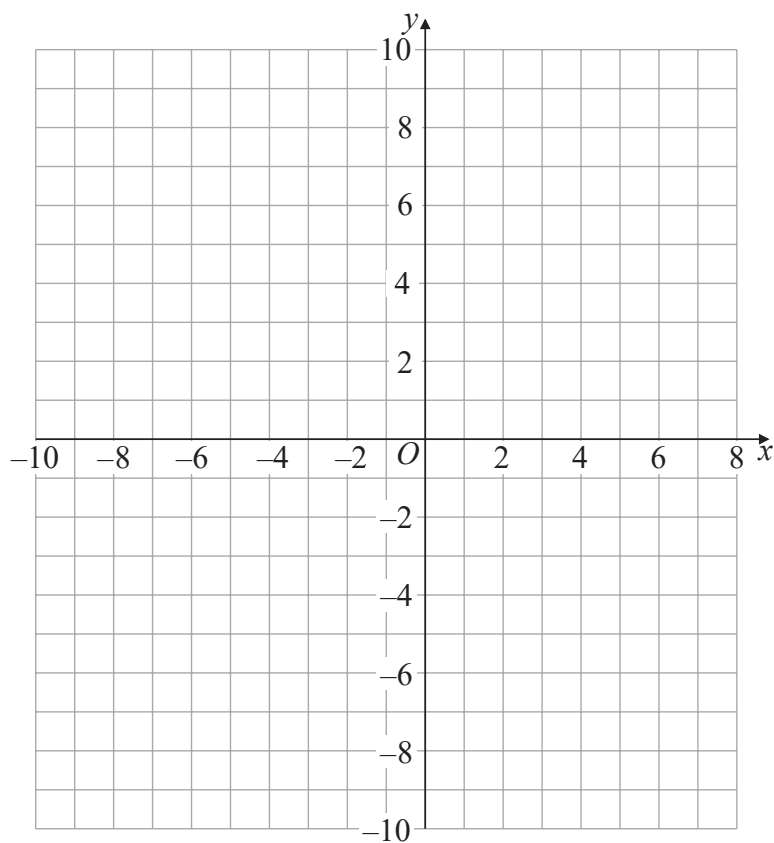
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Question 9 continued

Only use this grid if you need to redraw your triangles.



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(Total for Question 9 is 11 marks)



- 10 A rocket is launched from a point which is 5 m above horizontal ground. The rocket moves vertically upwards so that at time t seconds, the height, h metres, of the rocket above the ground is given by

$$h = 5 + 3t + 9t^2 - t^3$$

At time t seconds, the velocity of the rocket is v m/s and the acceleration of the rocket is a m/s²

- (a) Find an expression for v in terms of t . (2)
- (b) Find an expression for a in terms of t . (1)
- (c) Find the time when the rocket stops accelerating upwards. (2)

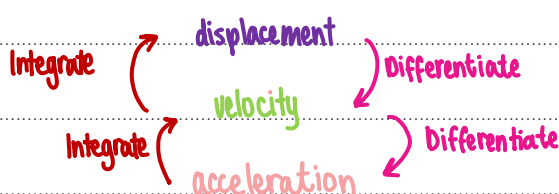
The rocket is instantaneously at rest when it is at point A .

- (d) Show that the height, in metres to one decimal place, of A above the ground is 131.2 m. (6)

The rocket now falls vertically downwards and hits the ground.

- (e) Find the total distance, to the nearest metre, travelled by the rocket at the instant it hits the ground. (2)

(a)



To find velocity using displacement, you need to differentiate.

The general rule for differentiation is "bring down the power and subtract 1"

$$\frac{dy}{dx} | ax^n = nax^{n-1}$$

$$\begin{aligned} \frac{dy}{dx} | 5 + 3t + 9t^2 - t^3 &= 3t^{1-1} + 2(9)t^{2-1} - (3)t^{3-1} \\ &= 3 + 18t - 3t^2 \end{aligned}$$

$$v = 3 + 18t - 3t^2$$

(b) To find acceleration, you need to differentiate velocity

$$\frac{dy}{dx} | 3 + 18t + 3t^2 = 18t^{1-1} - (2)3t^{2-1} = 18 - 6t$$

$$a = 18 - 6t$$

$$\left[\text{Solutions of } ax^2 + bx + c = 0 \text{ are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right]$$



Question 10 continued

(c) When the rocket stops accelerating, acceleration is equal to 0.

$$18 - 6t = 0$$

$$-6t = -18$$

$$t = 3$$

(d) At point A, the velocity of the rocket is 0 ms^{-1} . You can use this to find time.

$$3 + 18t - 3t^2 = 0$$

You can use the quadratic equation above to find t .

where $a = -3$ $t_1 = \frac{-(18) + \sqrt{(18)^2 - 4(-3)(3)}}{2(-3)}$ $t_2 = \frac{-(18) + \sqrt{(18)^2 - 4(-3)(3)}}{2(-3)}$

$$b = 18$$

$$c = 3$$

$$t_1 = 3 + \sqrt{10}$$

$$= 6.16...$$

$$t_2 = 3 - \sqrt{10}$$

$$= -0.16...$$

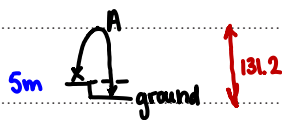
Time cannot be negative so you have to use the value of t , and substitute into the equation for h to find height.

$$h = 5 + 3(3 + \sqrt{10}) + 9(3 + \sqrt{10})^2 - (3 + \sqrt{10})^3$$

$$h = 131.245...$$

$$h \approx 131.2 \text{ m}$$

(e) Imagine X is the rocket.



Total distance: $131.2 + (131.2 - 5)$

$$= 257.4$$

$$\approx 257 \text{ m (nearest m)}$$



Question 10 continued

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Question 10 continued

Handwriting practice area with 20 horizontal dotted lines.

(Total for Question 10 is 13 marks)



11

$$f(x) = 4x^3 - 13x - 6$$

- (a) Use the factor theorem to show that $(2x + 1)$ is a factor of $f(x)$.

(2)

- (b) Hence factorise $f(x)$ fully.

(4)

The curve C has equation $y = f(x)$

- (c) Find the coordinates of the points of intersection of C with the x -axis.

(2)

- (d) Find the coordinates, to 2 decimal places, of the turning points of C .

(5)

The table below gives the coordinates of three points on C .

x	-2	0.5	1.5
y	-12	-12	-12

- (e) On the grid opposite, draw the curve C for $-2 \leq x \leq 2$
Clearly label the coordinates of the turning points of C and the coordinates of the points of intersection with the x -axis and the y -axis.

(3)

(a) When $2x + 1 = 0$

$$2x = -1$$

$$x = -\frac{1}{2}$$

Substitute into $f(x) \rightarrow 4\left(-\frac{1}{2}\right)^3 - 13\left(-\frac{1}{2}\right) - 6 = 0$

(c) To factorise $f(x)$, you can use long division or

Long division:

$$\begin{array}{r}
 2x^2 - x - 6 \\
 2x + 1 \overline{) 4x^3 - 13x - 6} \\
 \underline{-4x^3 + 2x^2} \\
 -2x^2 - 13x \\
 \underline{-(-2x^2 - x)} \\
 -12x - 6 \\
 \underline{-(-12x - 6)} \\
 0
 \end{array}$$

Synthetic Division when $x = -\frac{1}{2}$

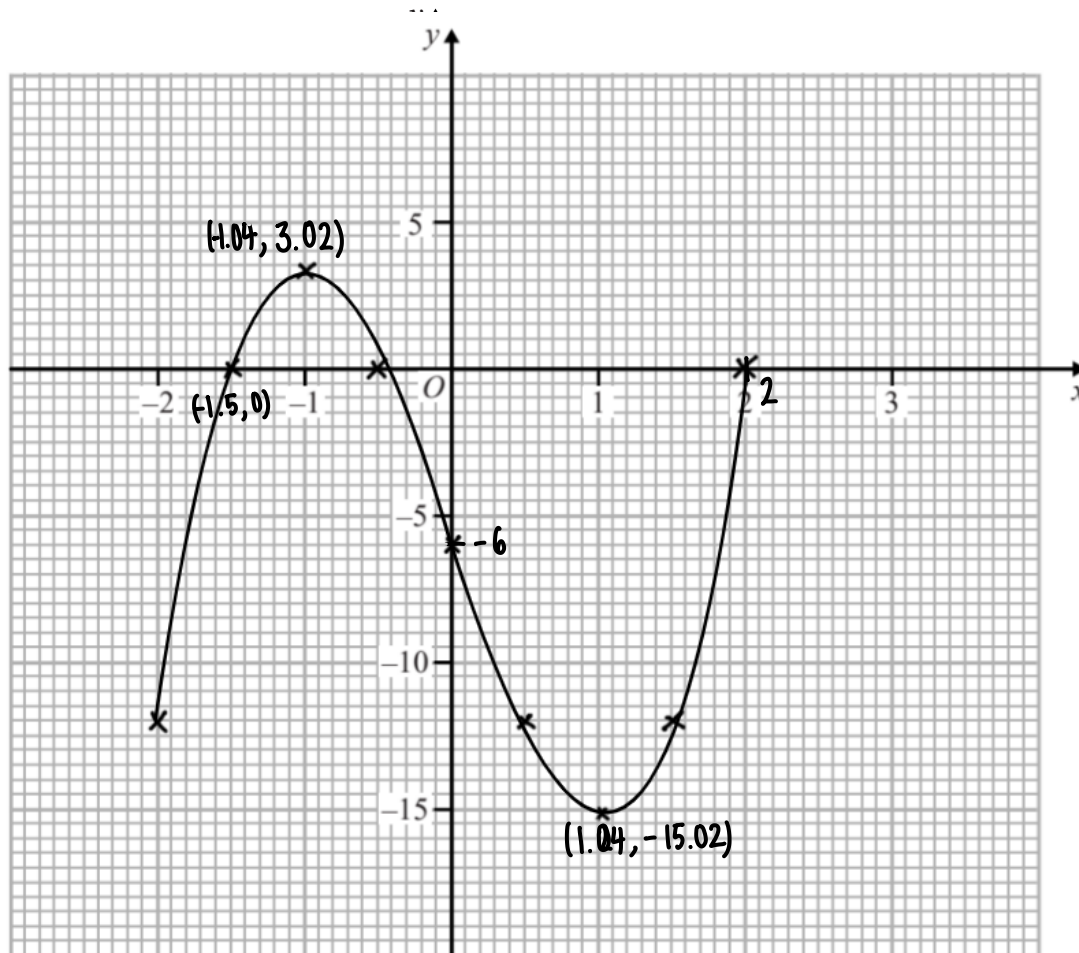
$$\begin{array}{r|rrrr}
 -\frac{1}{2} & 4 & 0 & -13 & -6 \\
 & & -2 & 1 & 6 \\
 \hline
 & 4 & -2 & -12 & 0 \\
 \end{array}$$

$\div 2 \rightarrow$

$$\begin{array}{r}
 2 \\
 2 - x - 6
 \end{array}$$



Question 11 continued



You can now factorise $2x^2 - x - 6$

method 1:

① Multiply the first and last numbers together.

$$2 \times -6 = -12$$

② What multiplies to make -12 and adds to make -1.

3 and -4

③ Cancel out terms in one bracket

$$\begin{aligned} & \overset{-2}{\cancel{2}}x \overset{-12}{\cancel{-4}}(2x+3) \\ & = (x-2)(2x+3) \end{aligned}$$

method 2:

① Multiply the first and last numbers together.

$$2 \times -6 = -12$$

② What multiplies to make -12 and adds to make -1.

3 and -4

③ Split middle term into the two numbers found above

$$2x^2 - 4x + 3x - 6$$

④ Take out what is common from each side

$$2x(x-2) + 3(x-2)$$

⑤ Place the terms in front of the brackets in another bracket

$$(2x+3)(x-2)$$

Turn over for a spare grid if you need to redraw your curve.



Question 11 continued

$$\therefore f(x) = (2x+1)(2x+3)(x-2)$$

(c) When the curve C intersects with the x-axis, $f(x) = 0$.

$$(2x+1)(2x+3)(x-2) = 0$$

$$2x+1=0$$

$$2x+3=0$$

$$x-2=0$$

$$2x = -1$$

$$2x = -3$$

$$x = 2$$

$$x = -\frac{1}{2}$$

$$x = -\frac{3}{2}$$

$$\left(-\frac{1}{2}, 0\right)$$

$$\left(-\frac{3}{2}, 0\right)$$

$$(2, 0)$$

(d) At the turning points, the gradient of the curve is 0. As such, you can use differentiation to find these turning points.

The general rule for differentiation is "bring down the power and subtract 1 from the power"

$$f'(x) = (3)4x^{3-1} - (1)13x^{1-1} - (0)6x^0$$

$$f'(x) = 12x^2 - 13$$

$$12x^2 - 13 = 0$$

$$12x^2 = 13$$

$$x^2 = \frac{13}{12}$$

$$x = \pm \sqrt{\frac{13}{12}}$$

$$x = \pm 1.040833$$

$$x \approx \pm 1.04 \text{ (2 dp)}$$

You can substitute the x-values into the original equation to find the y values.

$$y_1 = 4(1.408\dots)^3 - 13(1.408\dots) - 6$$

$$= -15.0205\dots$$

$$\approx -15.02 \text{ (2 dp)}$$

$$y_2 = 4(-1.04\dots)^3 - 13(-1.04\dots) - 6$$

$$= 3.0205\dots$$

$$\approx 3.02$$

Turning points: $(1.04, -15.02)$, $(-1.04, 3.02)$

(e) On graph above.



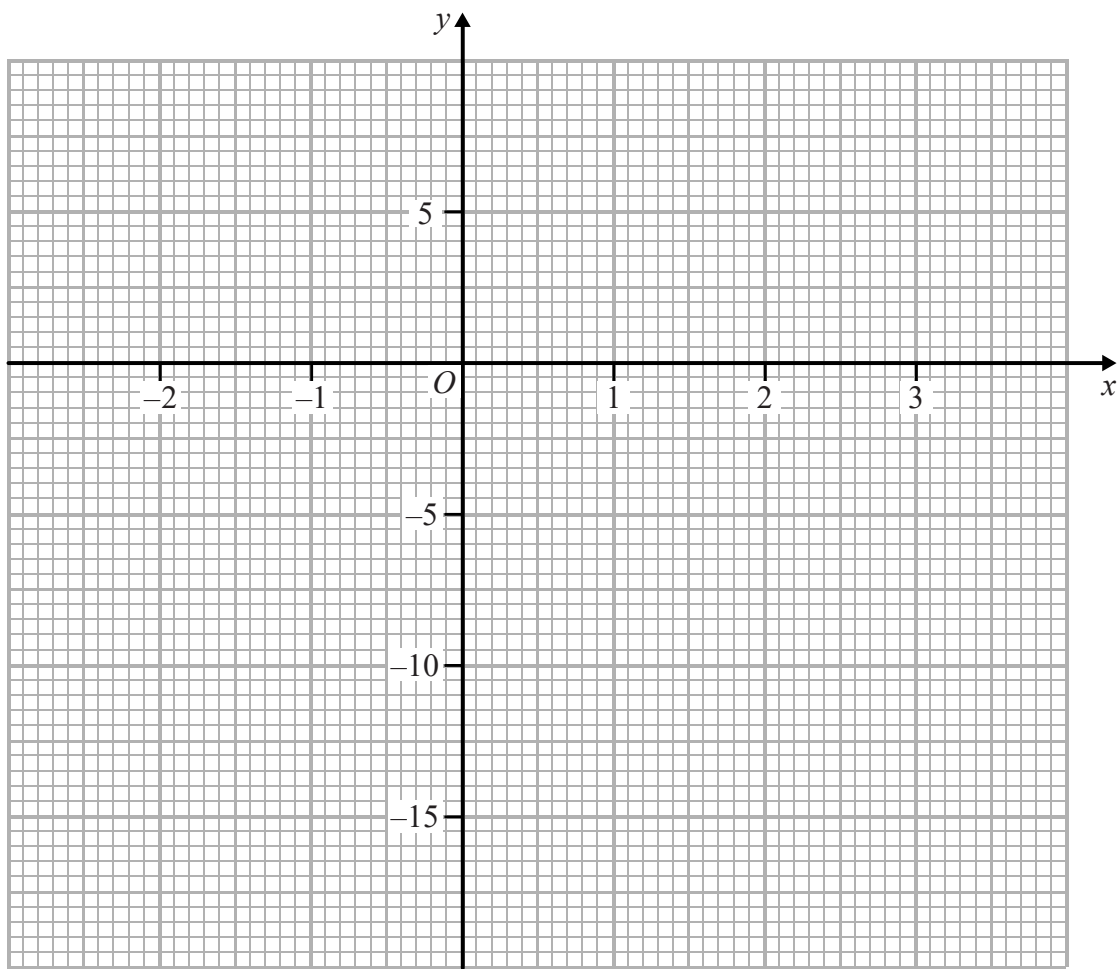
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Question 11 continued

Only use this grid if you need to redraw your curve.



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(Total for Question 11 is 16 marks)



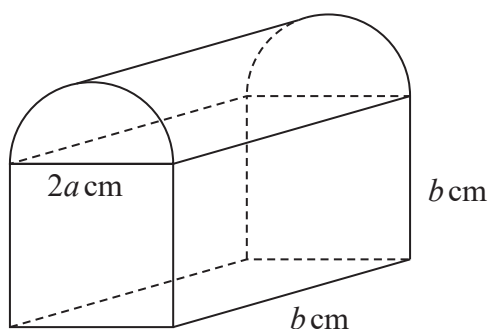


Diagram NOT
accurately drawn

Figure 3

Figure 3 shows a solid silver paperweight made from a cuboid and a half cylinder. The cuboid is $2a$ cm wide, b cm long and b cm high. The plane face of the half cylinder coincides with the top face of the cuboid. The total surface area of the paper weight is A cm²

- (a) Find an expression for A in terms of π , a and b .

(2)

Given that $a = 6\sqrt{5}$ and that the surface area of the paperweight can be written as

$$(2b^2 + 6ab + 60\pi\sqrt{15}) \text{ cm}^2$$

- (b) show that the exact value of b is $10\sqrt{3} - 6\sqrt{5}$

(5)

The paperweight is melted down to form a different cuboid. This second cuboid is $2a$ cm wide, b cm long and h cm high, as shown in Figure 4.

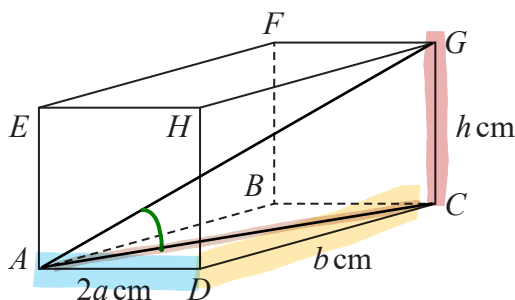


Diagram NOT
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Figure 4

- (c) Calculate the size, to the nearest degree, of angle GAC .

(5)

(a) The surface area is the sum of areas of all subjects.

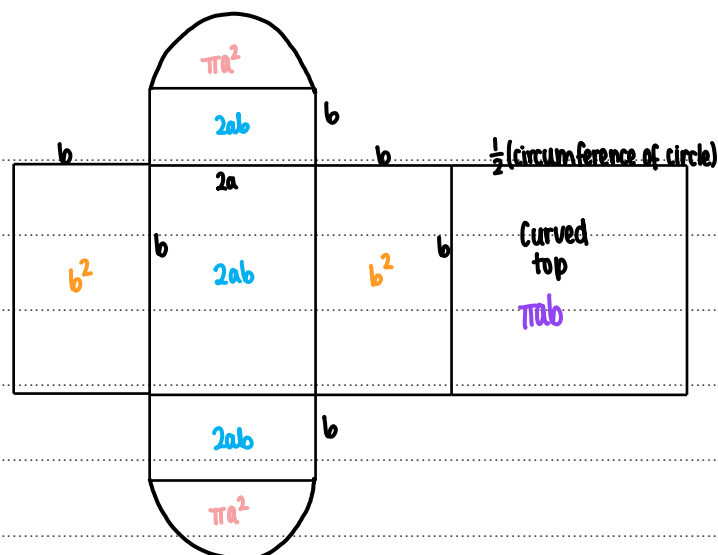
Area of a square/rectangle: length $\times$ width

Area of a circle: πr^2

It will be helpful to think of the shape in terms of its plan:



Question 12 continued



To find the area of the curved region, you need to multiply the circumference of the semicircle by the height, b .

Curved surface area of cylinder: $2\pi rh$

$$2 \times \pi \times a \times b = 2\pi ab$$

Curved surface area of half a cylinder: πab

Area of semicircles:

$$\frac{1}{2} \times \pi (a)^2 = \frac{1}{2} \pi a^2 \rightarrow \text{multiply by } \frac{1}{2} \text{ since it is a semicircle.}$$

$$\frac{1}{2} \pi a^2 + \frac{1}{2} \pi a^2 = \pi a^2$$

Surface Area:

$$2b^2 + 3(2ab) + 2\pi a^2 + \pi ab = 2b^2 + 6ab + \pi a^2 + \pi ab$$

(b) You need to substitute $a=6\sqrt{5}$ into your equation for Surface Area and the equation given, then solve simultaneously.

$$2b^2 + 6(6\sqrt{5})b + \pi(6\sqrt{5})^2 + \pi(6\sqrt{5})b = 2b^2 + 6(6\sqrt{5})b + 60\pi\sqrt{5}$$

$$2b^2 + 36\sqrt{5}b + 180\pi + 6\sqrt{5}b\pi = 2b^2 + 36\sqrt{5}b + 60\pi\sqrt{5}$$

$$180\pi + 6\sqrt{5}b\pi = 60\pi\sqrt{5}$$

$$6\sqrt{5}b\pi = 60\pi\sqrt{5} - 180\pi$$

$$b = \frac{60\sqrt{5} - 180}{6\sqrt{5}}$$

$$b = \frac{6(10\sqrt{5} - 30)}{6\sqrt{5}}$$

$$\left[\begin{array}{l} \text{Volume of cylinder} = \pi r^2 h \\ \text{Curved surface area of cylinder} = 2\pi rh \end{array} \right]$$

Question 12 continued

$$b = \frac{10\sqrt{15} - 30}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \quad \text{Rationalise the denominator}$$

$$= \frac{10\sqrt{15} \times \sqrt{5} - 30\sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

$$= \frac{10\sqrt{75} - 30\sqrt{5}}{5}$$

$\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$

$$= \frac{10 \times 5\sqrt{3} - 30\sqrt{5}}{5}$$

Divide by 5

$$= 10\sqrt{3} - 6\sqrt{5}$$

(c) We know the value of $a = 6\sqrt{5}$ and $b = 10\sqrt{3} - 6\sqrt{5}$ but we need to find the value of h .

Although, the paperweight's shape has changed, its volume is the same. Hence we need to find the volume of both to calculate the value h .

Volume of cuboid₁ and half cylinder:

Volume of cuboid = length $\times$ width $\times$ height

$$\text{Volume of cuboid}_1 = 2a \times b \times b$$

$$= 2ab^2$$

Volume of half cylinder:

$$\text{Volume of cylinder: } \pi r^2 h$$

$$\text{Volume of } \frac{1}{2} \text{ cylinder: } \frac{1}{2} \times \pi \times a^2 \times b$$

$$= \frac{1}{2} \pi a^2 b$$

$$\text{Volume of cuboid}_1 \text{ and half cylinder: } 2ab^2 + \frac{1}{2} \pi a^2 b$$

$$\text{Volume of cuboid}_2: 2a \times b \times h$$

$$= 2abh$$



Question 12 continued

Volume of cuboid₁ and half cylinder = Volume of cuboid₂

$$\begin{aligned}
 2ab^2 + \frac{1}{2}\pi a^2b &= 2abh \\
 \div b &\rightarrow 2ab + \frac{1}{2}\pi a^2 = 2ah \\
 \div a &\rightarrow 2b + \frac{1}{2}\pi a = 2h \\
 \div 2 &\rightarrow b + \frac{1}{4}\pi a = h
 \end{aligned}$$

You can now substitute your known values to find h :

$$h = (10\sqrt{3} - 6\sqrt{5}) + \frac{1}{4} \times \pi \times (6\sqrt{5})$$

$$h = 14.441\dots$$

$$\approx 14.4$$

To find $\angle GAC$ using $\sin^{-1}$ or $\tan^{-1}$, you need to know the values of line AC and GC .

$$GC = h = 14.441\dots$$

You can find line AC using Pythagoras' theorem:

$$a^2 + b^2 = c^2$$

$$AD^2 + CD^2 = AC^2$$

$$(2a)^2 + b^2 = AC^2$$

$$(2(6\sqrt{5}))^2 + (10\sqrt{3} - 6\sqrt{5})^2 = AC^2$$

$$\sqrt{(2(6\sqrt{5}))^2 + (10\sqrt{3} - 6\sqrt{5})^2} = AC$$

$$27.115\dots = AC$$

Since you have found the opposite and adjacent lengths, you can use $\tan^{-1}$ to find the angle

$$\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan \angle GAC = \frac{14.44\dots}{27.11\dots}$$

$$\angle GAC = \tan^{-1} \left(\frac{14.44\dots}{27.11\dots} \right)$$

$$= 28.04\dots$$

$$\approx 28^\circ$$

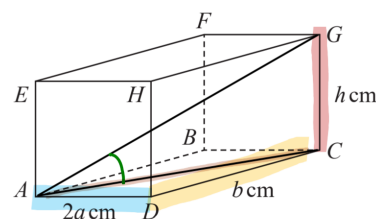


Figure 4

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TOTAL FOR PAPER IS 100 MARKS



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